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Operating-Point Selection for Linearized Power-Flow Models in Active Distribution Grids: Accuracy, Critical-State Performance, and Runtime

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Abstract

Active distribution grids require analysis methods that combine physical fidelity with low computational cost for repeated screening, optimization workflows, and operational decision support. Local linearized power-flow models can support these tasks, but their accuracy depends strongly on the operating point around which they are derived and on how the relevant operating range is represented. This paper benchmarks operating-point-dependent linearized power-flow models for active distribution grids across five SimBench networks. We compare operating-point selection strategies, library sizes, and targeted library extensions for three separate target quantities: voltage magnitude, line loading, and transformer loading. The evaluation combines equal-budget accuracy, critical- and near-limit operating states, post-action AC validation, and controlled runtime measurements. At an equal budget of 36 linearization points, k-medoids provides the most consistent general-purpose accuracy and achieves the first target-specific rank for all three quantities. Increasing the library size yields diminishing gains, with the largest improvement between 10 and 22 points. A k-medoids/k-center hybrid improves all six primary accuracy metrics relative to a common 28-point basis and provides the best upper-tail voltage accuracy in critical states. In post-action validation, nominal linear actions are AC-feasible in 31 of 35 cases, while adding a safety margin yields successful AC results in all 35 cases without new violations. Online evaluation requires about 3.5 ms per state and achieves a median speed-up of about 26–28 relative to AC power flow. The results show that operating-point libraries can provide a practical accuracy–runtime compromise when representative coverage, critical-state validation, and residual decision margins are considered jointly.

Keywords: active distribution grids; linearized power flow; operating-point selection; local linearization; model libraries; critical states; post-action validation; runtime–accuracy trade-off



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1. Introduction

Distribution grids are increasingly operated under conditions that are both more variable and more controllable than in conventional passive-grid operation. Photovoltaic generation, electric vehicles, heat pumps, battery storage systems, and other flexible devices change local loading patterns and can move voltage and loading constraints closer to their technical limits. At the same time, distribution-system operators and planning tools require repeated analyses over many time steps, scenarios, and candidate actions.

This creates a trade-off between physical fidelity and computational effort: nonlinear AC power-flow calculations provide the physical reference, but repeated AC evaluations can become computationally expensive when embedded in screening, time-series simulation, or optimization workflows [1–3].

Linearized power-flow models reduce this computational burden by replacing, approximating, or locally simplifying the nonlinear AC power-flow equations while preserving information about voltages and network constraints for the task at hand. Classical distribution-grid approximations were strongly influenced by radial branch-flow formulations and the DistFlow model family [4]. Later work extended this line to convex relaxations and linear approximations for multiphase radial networks [5], explicit linear power-flow models for multiphase distribution systems [6], and generalized LinDistFlow formulations that can be linearized around arbitrary operating points [7]. Related approaches derive linear approximations from existence and fixed-point analyses of distribution-network power flow [8], from implicit linearization of the power-flow manifold [9], or from rectangular-coordinate perturbation models [10]. Sensitivity-based formulations are particularly relevant for operational screening because they directly relate changes in active and reactive injections to voltage and loading-related quantities [11–13].

The accuracy of such models depends on the operating point around which they are derived. A first-order approximation is most reliable near its reference state and may deteriorate when injections, demands, or flexibility activations move the grid into a different region of the operating space. This issue has motivated several approaches for wider operating ranges, including sample-based conservative approximations [14], adaptive approximations using second-order sensitivity information [15], sample-based piecewise linear approximations [16], and parameter-optimized LinDistFlow-type models [17]. These studies show that approximation quality is shaped by both the algebraic form of the linear model and the representation of the relevant operating range.

Recent work illustrates that these two design dimensions can be addressed in different ways. Di Fazio et al. incorporate voltage-control devices into a radial-distribution linear power-flow formulation [18]; Sulaimon et al. combine two linear load-flow formulations in a hybrid model [19]; and Taheri et al. optimize the parameters of LinDistFlow-type approximations over operating samples [17]. The present study fixes one AC-Jacobian-based local model form and varies only how its operating-point library is constructed and assigned.

The resulting selection problem is closely related to the use of representative periods in energy-system and power-system studies, where long time series are reduced to a limited set of characteristic days or periods to retain relevant variability at tractable computational cost [20–23]. These studies also show that representative selections may underrepresent rare or extreme events unless such states are treated explicitly [24]. For operating-point-dependent power-flow linearization, the selected objects are grid states used to construct AC-reference solutions and local sensitivity matrices. This differs from the representative days commonly used in planning studies. Nevertheless, the same distinction between typical representativeness and coverage of remote or critical states is central. In clustering terms, k-medoids emphasizes representative observed samples [25], whereas k-center-type selection emphasizes coverage of the largest assignment distance [26]. Operational use requires additional validation beyond average approximation error. A model used for screening or decision support must behave reliably in states close to voltage and loading limits, where small residual errors can change a constraint classification. Related work on voltage-violation detection and sensor placement highlights the importance of operationally relevant critical states [27], while feasibility-restoration approaches show that approximate or optimized power-flow solutions may require AC validation or correction

before they can be treated as feasible [28]. Online-updated linear models and feedback-based optimization address model mismatch during operation [2,29]. For flexibility-enabled operational optimization, reusable local model libraries therefore require a clear assessment of accuracy, critical-state behavior, post-action validity, and runtime.

This paper addresses this model-library problem. It evaluates how different operating-point libraries affect the practical usefulness of operating-point-dependent local linearizations for active distribution grids. The comparison is carried out on a reproducible SimBench benchmark [30] and treats voltage magnitude, line loading, and transformer loading as separate target quantities. The investigated libraries include statistically representative, coverage-oriented, structured seasonal, and hybrid selections. This separation is important because a library that performs well for one target quantity or error region need not be equally effective for another, and because operational use requires reliable behavior near technical limits and after actions derived from the linear model.

The study is organized around the following research question: how should operating-point libraries for local linearized power-flow models be selected so that they provide a useful accuracy–runtime compromise for repeated distribution-grid analysis, critical-state assessment, and action evaluation? Five aspects are evaluated. First, the paper compares operating-point selection strategies at an equal budget of 36 linearization points. Second, it studies how the number of representative points affects marginal accuracy gains. Third, it evaluates targeted extensions of a common base library to distinguish broad statistical representativeness from additional coverage of difficult regions. Fourth, it analyzes critical and near-limit operating states and validates selected linear actions by AC power flow after the predicted intervention has been applied. Fifth, it measures offline and online runtime under controlled conditions.

The main contributions are therefore as follows. First, the paper provides a consistent benchmark of operating-point-dependent local linearizations across five SimBench networks and three monitored target quantities. Second, it compares k-medoids, k-center, structured seasonal, and hybrid operating-point libraries under a common validation protocol. Third, it quantifies the effect of library size using 10, 22, 36, and 50 linearization points and separates marginal accuracy gains from the runtime–accuracy trade-off. Fourth, it evaluates approximation quality in critical and near-limit states rather than relying only on average time-series errors. Fifth, it tests whether linear actions remain valid after they are applied to the nonlinear AC model and assesses how an explicit safety margin changes post-action feasibility.

The remainder of the paper is structured as follows. Section 2 introduces the AC-reference formulation and the local linearization framework. Section 3 defines the operating-point libraries and assignment rules used in the study. Section 4 describes the benchmark data, validation sample, error metrics, critical-state definition, post-action validation, and runtime protocol. Section 5 reports the numerical results. Section 6 discusses their implications, limitations, and transferability. Section 7 concludes the paper.

2. Grid Calculation and Linearization Fundamentals

This section defines the grid-calculation and linearization framework used in the remainder of the paper. It establishes a common notation for the AC reference, the local first-order approximation, and the operating-point-dependent model libraries evaluated in the benchmark. The formulation is deliberately written in terms of generic inputs and outputs so that voltage magnitude, line loading, and transformer loading can be treated within one consistent framework. The notation follows standard implicit AC power-flow and first-order sensitivity formulations used in linearized and sensitivity-based

distribution-grid models [1,6,8,9,11,13]; the derivation is restated here to make the target quantities and library assignment explicit for the present benchmark.

2.1. AC Power-Flow Formulation

The steady-state operation of a distribution grid is described by the nonlinear AC power-flow equations. In compact implicit form, the grid model is written as

$$g(x, u) = 0, \quad (1)$$

where x denotes the vector of network state variables and u denotes the vector of nodal active and reactive power injections. In this study, u collects net nodal injections resulting from load, generation, and flexibility-related injection changes. The state vector x contains the variables required by the AC power-flow model, such as voltage magnitudes and voltage angles.

For a given injection vector u , solving the AC power-flow problem means finding a state vector x that satisfies Equation (1). The corresponding AC solution is used as the nonlinear reference throughout the study. All approximation errors reported later are therefore deviations between a linearized estimate and an AC reference for the same network, time step, and injection vector. This convention is important because the paper compares alternative operating-point representations rather than alternative nonlinear reference models.

2.2. Local Linearization Around an Operating Point

Let $(x^{(h)}, u^{(h)})$ denote a feasible operating point, where h indexes the operating point in a library of local models. The injection vector $u^{(h)}$ defines the nodal operating condition, and $x^{(h)}$ is the corresponding AC power-flow solution. Hence,

$$g(x^{(h)}, u^{(h)}) = 0. \quad (2)$$

For a nearby operating state, deviations from this operating point are defined as

$$\Delta x = x - x^{(h)}, \quad \Delta u = u - u^{(h)}. \quad (3)$$

A first-order Taylor expansion of Equation (1) around $(x^{(h)}, u^{(h)})$ yields

$$g(x, u) \approx g(x^{(h)}, u^{(h)}) + J^{(h)} \Delta x + G^{(h)} \Delta u, \quad (4)$$

where

$$J^{(h)} = \left. \frac{\partial g}{\partial x} \right|_{(x^{(h)}, u^{(h)})}, \quad G^{(h)} = \left. \frac{\partial g}{\partial u} \right|_{(x^{(h)}, u^{(h)})} \quad (5)$$

are the Jacobian matrices with respect to the state variables and nodal injections. Since the expansion point is AC-feasible, the zero-order term vanishes and the local relation becomes

$$J^{(h)} \Delta x + G^{(h)} \Delta u \approx 0. \quad (6)$$

If $J^{(h)}$ is nonsingular, Equation (6) can be solved for the state deviation:

$$\Delta x \approx - \left(J^{(h)} \right)^{-1} G^{(h)} \Delta u. \quad (7)$$

The matrix product in Equation (7) is evaluated at operating point h . The approximation is therefore local: its accuracy depends on how far the evaluated state is from the

operating point used to derive the Jacobian matrices and on how nonlinear the relevant output quantities are in that region.

2.3. Sensitivity Matrices for Target Quantities

Operational assessment usually concerns output quantities derived from the AC solution rather than the state vector itself. In this paper, the evaluated target quantities are nodal voltage magnitude, line loading, and transformer loading. These quantities are collected in an output vector

$$y = \psi(x, u), \quad (8)$$

where $\psi(\cdot)$ maps the AC state and the nodal injections to the selected monitored quantities. This notation also covers intermediate flow quantities if they are required to compute loading values, but the benchmark results are reported for voltage magnitude, line loading, and transformer loading separately.

A first-order Taylor approximation of Equation (8) around $(x^{(h)}, u^{(h)})$ gives

$$\Delta y \approx C_x^{(h)} \Delta x + C_u^{(h)} \Delta u, \quad (9)$$

with

$$C_x^{(h)} = \left. \frac{\partial \psi}{\partial x} \right|_{(x^{(h)}, u^{(h)})}, \quad C_u^{(h)} = \left. \frac{\partial \psi}{\partial u} \right|_{(x^{(h)}, u^{(h)})}. \quad (10)$$

Substituting Equation (7) into Equation (9) yields the local input–output approximation

$$\Delta y \approx \left(C_u^{(h)} - C_x^{(h)} \left(J^{(h)} \right)^{-1} G^{(h)} \right) \Delta u = H^{(h)} \Delta u. \quad (11)$$

The operating-point-dependent sensitivity matrix is therefore

$$H^{(h)} = C_u^{(h)} - C_x^{(h)} \left(J^{(h)} \right)^{-1} G^{(h)}. \quad (12)$$

Each row of $H^{(h)}$ corresponds to one monitored output quantity, and each column corresponds to one component of the injection vector. The same construction is used for the three target-specific output vectors evaluated in the study. For voltage magnitude,

$$\Delta v \approx H_v^{(h)} \Delta u, \quad (13)$$

where v denotes nodal voltage magnitudes. For thermal loading, the corresponding local approximation is written as

$$\Delta \ell \approx H_\ell^{(h)} \Delta u, \quad (14)$$

where ℓ denotes line or transformer loading in percent of the respective equipment rating. Loading is derived from apparent power and equipment ratings; the relation in Equation (14) should therefore be understood as a local approximation of the loading response around the operating point.

If active and reactive branch-flow deviations are evaluated explicitly, they can be represented by

$$\Delta f \approx H_f^{(h)} \Delta u, \quad (15)$$

where f denotes the corresponding flow vector. In the present benchmark, such flow-related sensitivities serve primarily as part of the general linearization framework and as an intermediate basis for loading-related quantities; the reported primary accuracy metrics remain target-specific for voltage magnitude, line loading, and transformer loading.

The sensitivity matrices used in the benchmark are computed analytically at each converged AC operating point; numerical finite-difference perturbations are not used. Specifically, the reduced polar-coordinate AC power-flow Jacobian provided by the power-flow solver is inverted and partitioned into the four blocks $\partial\theta/\partial P$, $\partial\theta/\partial Q$, $\partial V/\partial P$, and $\partial V/\partial Q$. These blocks are mapped from the solver's internal ordering to the complete network-bus ordering. The implementation assumes one slack bus, no voltage-controlled PV buses, and PQ representation of all remaining buses, as applicable to the investigated SimBench models. Voltage-magnitude sensitivities are obtained directly from the corresponding Jacobian-inverse blocks. For lines and transformers, the voltage-magnitude and angle sensitivities are propagated through the complex terminal-current equations; the complex-current changes are projected onto the operating-point current magnitude and divided by the respective thermal rating to obtain loading changes in percentage points. Thus, voltage, line-loading, and transformer-loading sensitivities are all derived from the same analytical AC-Jacobian evaluation at the assigned operating point.

2.4. Operating-Point-Dependent Model Libraries

A local linear model is tied to the operating point at which its sensitivity matrix is derived. If a full time series is represented by a single operating point, all time steps are approximated with the same matrix,

$$\Delta y_t \approx H^{(1)} \Delta u_t, \quad (16)$$

where t denotes the time step and $H^{(1)}$ is derived at one fixed reference state. This simple formulation cannot represent a wide operating range with uniform accuracy.

The evaluated approach instead uses a library of operating-point-dependent local models. Let $h^*(t)$ denote the library element assigned to time step t . The local deviation model is then

$$\Delta y_t \approx H^{(h^*(t))} (u_t - u^{(h^*(t))}), \quad (17)$$

or, including the output value at the assigned operating point,

$$\hat{y}_t = y^{(h^*(t))} + H^{(h^*(t))} (u_t - u^{(h^*(t))}). \quad (18)$$

Equations (17) and (18) describe the common model form used in the subsequent benchmark. Different methods differ in how the library points are selected, how many local models are included, and how evaluated states are assigned to those models. The central methodological question is therefore how to represent the relevant operating range so that the resulting library remains accurate for typical states, critical states, and post-action states while retaining the computational advantage of linear evaluation.

3. Operating-Point Library Construction

The local linearization framework in Section 2 defines a linear model around one operating point. For time-series analysis in active distribution grids, however, no single state can represent the full range of photovoltaic feed-in, load demand, and flexible consumption. This section therefore defines the operating-point libraries used in this study. An operating-point library is a finite set of representative grid states for which AC-reference solutions and local sensitivity matrices are constructed. During application, each evaluated state is assigned to one representative state, and the corresponding local model is used.

The purpose of this section is methodological rather than numerical. It describes how the operating range is represented before Section 4 defines the benchmark data, validation states, and error metrics. The final evaluation focuses on five library families: k-medoids

libraries, k-center libraries, structured seasonal libraries, k-medoids/k-center hybrids, and seasonal hybrids. These libraries are compared for three target quantities: voltage magnitude, line loading, and transformer loading.

3.1. Operating-Point Libraries and Assignment Rule

Let T denote the set of candidate time steps from which representative operating points may be selected. For the distance-based selection procedures, T comprises a common deterministic pool of 1000 time steps selected at approximately uniform intervals across the annual time series. Each candidate state is represented by the unscaled bus-wise net active-power vector $z_t \in \mathbb{R}^m$, with one component for every network bus:

$$z_{t,b} = -p_{t,b}^{\text{load}} + p_{t,b}^{\text{sgen}} + p_{t,b}^{\text{gen}} - p_{t,b}^{\text{storage}}, \quad b = 1, \dots, m, \quad (19)$$

where positive values denote net generation and negative values denote net demand; all components are expressed in MW. No reactive-power features, dimensionality reduction, or feature normalization are applied. Selection and assignment use the Euclidean distance

$$d(z_t, z_s) = \|z_t - z_s\|_2. \quad (20)$$

This deterministic subsample reduces pairwise-distance computation and memory use, particularly for k-medoids, while retaining year-wide coverage. The restriction applies only to offline library construction; all methods are subsequently evaluated on the common validation sample of 500 time steps per network defined in Section 4.2.

The active-power-only representation reflects the fixed power-factor assumptions of the investigated SimBench grids: 1.0 for distributed energy resources and 0.93 for individual loads and storage units [30]. At component level, reactive-power variations therefore follow the corresponding active-power profiles and provide limited independent temporal information in this benchmark. The selected representation is consequently a benchmark-specific simplification and requires reassessment for grids with independent reactive-power profiles, compensation devices, or voltage-dependent controls such as $Q(U)$.

The distance in Equation (20) is used as a scalable, target-independent proxy for similarity between operating states. Separate bus coordinates retain the location of an injection difference, but the Euclidean norm weights equal active-power differences at all buses equally and does not include the target- and operating-point-dependent sensitivities in $H^{(h)}$. Consequently, two state differences with the same feature-space norm can produce different voltage or loading changes through $\Delta y \approx H^{(h)} \Delta u$. The adequacy of the proxy is therefore assessed empirically and separately for each target quantity.

An operating-point library with K representatives is defined by an index set

$$M = \{m_1, \dots, m_K\} \subseteq T. \quad (21)$$

For each selected index m_h , an AC power flow is solved and a local linear model is constructed around the corresponding injection vector $u^{(h)} = u_{m_h}$. The resulting library can be written as

$$\mathcal{L} = \left\{ \left(u^{(h)}, y^{(h)}, H^{(h)} \right) \middle| h = 1, \dots, K \right\}, \quad (22)$$

where $y^{(h)}$ denotes the AC-reference output at the representative state and $H^{(h)}$ is the corresponding output sensitivity matrix.

For an evaluated time step t , the assigned local model is selected by nearest-representative assignment in the same unscaled bus-wise active-power feature space:

$$h^*(t) = \arg \min_{h \in \{1, \dots, K\}} d(z_t, z_{m_h}), \quad (23)$$

where $d(\cdot, \cdot)$ is the Euclidean distance defined in Equation (20). The assigned representative $h^*(t)$ determines which local sensitivity matrix is used in the output approximation defined in Equation (18). All library variants in this paper share this application principle; they differ only in how the index set M is constructed and, in the gated seasonal variant, when an extension point is allowed to replace the assignment from the base library.

3.2. *k*-Medoids Libraries

The *k*-medoids strategy selects representative operating points directly from the observed candidate states. This property is important for local power-flow linearization because every representative is an actual grid state with a well-defined AC solution. For a prescribed library size K , the medoid set is chosen to minimize the average assignment distance:

$$M^{\text{med}} = \arg \min_{M \subseteq T, |M|=K} \sum_{t \in T} \min_{m \in M} d(z_t, z_m). \quad (24)$$

This objective favors representatives that are central with respect to the empirical distribution of operating states. It therefore provides a natural default strategy when the intended use case is repeated evaluation over a broad time series.

In the final benchmark, *k*-medoids libraries serve three roles. First, the 36-point *k*-medoids library is the main equal-budget reference because it provides the strongest general-purpose performance in the final results. Second, *k*-medoids libraries with $K \in \{10, 22, 36, 50\}$ are used to study how approximation accuracy changes with library size. Third, a 28-point *k*-medoids library forms the common basis for the hybrid-ablation study. In all cases, the selected medoids remain feasible AC operating states and define complete local linear models.

3.3. *k*-Center Libraries and Coverage-Oriented Selection

Whereas *k*-medoids emphasizes average representativeness, *k*-center selection emphasizes coverage of the operating range. For a prescribed library size K , the ideal *k*-center problem can be written as

$$M^{\text{cen}} = \arg \min_{M \subseteq T, |M|=K} \max_{t \in T} \min_{m \in M} d(z_t, z_m). \quad (25)$$

The objective reduces the largest distance between any candidate state and its nearest representative. This makes *k*-center selection attractive for covering poorly represented regions of the operating space, including rare or extreme states. In the implementation used for the benchmark, the *k*-center set is constructed by a greedy farthest-point procedure in the unscaled bus-wise active-power feature space.

A standalone 36-point *k*-center library is included in the equal-budget comparison to test whether coverage-oriented selection is competitive with distribution-oriented selection. In addition, *k*-center points are used as targeted extensions of a *k*-medoids base library. This distinction matters because *k*-center selection may be too strongly driven by extremes when used as the only selection principle, but it can be valuable when added to a statistically representative basis.

The *k*-medoids and *k*-center objectives in Equations (24) and (25) are defined exclusively by distances in the common feature space and contain no voltage or loading error

terms. The benchmark therefore treats their alignment with the three separately evaluated target quantities as an empirical question.

3.4. Structured Seasonal Libraries

The structured seasonal strategy provides an interpretable alternative to purely distance-based statistical selection. Instead of optimizing a global distance objective, representative states are selected from predefined calendar and operating-regime categories. The final 36-point seasonal library combines four seasons with nine characteristic operating regimes per season:

$$M^{\text{sea}} = \{m(s, r) | s \in \mathcal{S}, r \in \mathcal{R}\}, \quad |\mathcal{S}| = 4, \quad |\mathcal{R}| = 9. \quad (26)$$

Within each season, the candidate time steps are first restricted to the time window of the respective regime. Minimum and maximum rules select the observed time step with the corresponding extremal value. For quantile rules, the observed time step whose indicator is closest to the specified quantile is selected. Here, $P_{\text{net}}(t) = \sum_b z_{t,b}$ denotes the total net active-power injection, $P_{\text{gen}}(t)$ the total active-power generation, and $P_{\text{load}}(t)$ the total active-power demand. Table 1 defines all nine regimes used in each of the four seasons.

Table 1. Characteristic operating regimes used to construct the structured seasonal library. Each rule is applied separately within all four seasons, yielding $4 \times 9 = 36$ observed representative states.

No.	Regime	Time Window	Indicator and Selection Rule
1	Night low load	00:00–04:00	Minimum of $ P_{\text{net}} $
2	Morning transition	04:00–08:00	Median ($q = 0.50$) of $ P_{\text{net}} $
3	Moderate morning	08:00–10:00	Upper quartile ($q = 0.75$) of $ P_{\text{net}} $
4	Late-morning PV peak	10:00–12:00	Maximum of P_{gen}
5	Midday PV peak	12:00–14:00	Maximum of P_{gen}
6	Moderate afternoon	14:00–16:00	Upper quartile ($q = 0.75$) of $ P_{\text{net}} $
7	Early-evening peak load	16:00–18:00	Maximum of P_{load}
8	Late-evening peak load	18:00–20:00	Maximum of P_{load}
9	Night transition	20:00–00:00	Median ($q = 0.50$) of $ P_{\text{net}} $

These regime rules are used only to select the seasonal representatives. During validation, an evaluated state is still assigned to its nearest seasonal representative according to Equations (20) and (23); the calendar category of the evaluated time step does not determine the assigned local model. Each selected representative is an observed state with a converged AC power flow.

This strategy is included because it is transparent and easy to communicate in practical workflows. Its construction is less adaptive to the empirical geometry of the operating-state distribution than k-medoids or k-center selection, but it provides a useful reference for assessing how far an expert-informed seasonal design can approximate a data-driven operating-point library under the same benchmark conditions.

3.5. Hybrid and Gated Extension Libraries

Hybrid libraries are used to separate two questions that would otherwise be confounded: how well a representative base library performs, and how much a targeted extension can improve this library. The common basis is a 28-point k-medoids library,

$$M^{\text{base}} = M_{28}^{\text{med}}. \quad (27)$$

This basis is then extended by eight additional representatives,

$$M^{\text{hyb}} = M^{\text{base}} \cup M^{\text{ext}}, \quad |M^{\text{ext}}| = 8, \quad |M^{\text{hyb}}| = 36. \quad (28)$$

Using the same 28-point basis for all hybrid variants ensures that differences between hybrids can be attributed to the extension strategy rather than to a completely different 36-point library.

Two extension principles are considered. The k-medoids/k-center hybrid adds coverage-oriented k-center representatives to the k-medoids basis. The seasonal hybrid adds representatives selected from the structured seasonal design. A third variant, the gated seasonal hybrid, uses the same idea of a seasonal extension but restricts when the extension can replace the assignment from the medoid basis. Formally, let $h_{\text{base}}^*(t)$ and $h_{\text{ext}}^*(t)$ denote the nearest assignments within the base and extension sets. The gated assignment can be written generically as

$$h_{\text{gate}}^*(t) = \begin{cases} h_{\text{ext}}^*(t), & \text{if } g(t) = 1, \\ h_{\text{base}}^*(t), & \text{otherwise,} \end{cases} \quad (29)$$

where $g(t)$ is a gating criterion that allows the extension assignment only for states for which the extension is intended to be active. The purpose of the gated variant is to test whether a targeted seasonal extension can be prevented from degrading states that are already well represented by the k-medoids basis.

For comparison, the hybrid analysis also includes a jointly selected 36-point k-medoids library. This library has the same total size as the hybrids, but its 36 representatives are selected simultaneously rather than by adding eight points to an existing 28-point basis. It therefore provides an equal-size reference for assessing whether targeted extension is preferable to a full reselection of the library.

3.6. Library Variants Used in the Final Evaluation

Table 2 summarizes the operating-point library variants used in the final evaluation. The table distinguishes the broad equal-budget comparison, the library-size analysis, and the controlled hybrid-ablation setting. All variants are evaluated with respect to the target quantities defined in Section 4: voltage magnitude, line loading, and transformer loading.

Table 2. Operating-point library variants and their role in the final evaluation.

Library Variant	Points	Selection Principle	Role in the Evaluation
k-medoids	10, 22, 36, 50	Distribution-oriented medoid selection from observed operating states	Default data-driven library; used for the point-count study and as the 36-point equal-budget reference.
k-center	36	Coverage-oriented greedy selection in unscaled active-power feature space	Tests whether maximum-distance coverage alone is competitive at the same library size.
Structured seasonal	36	Four seasons combined with nine characteristic operating regimes	Interpretable non-clustering reference based on recurring seasonal and daily operating situations.
k-center hybrid	28 + 8	28 k-medoids base points plus eight k-center extension points	Tests whether coverage-oriented points can improve a representative base library.
Seasonal hybrid	28 + 8	28 k-medoids base points plus eight seasonal extension points	Tests whether interpretable seasonal representatives improve the same base library.
Gated seasonal	28 + 8	Seasonal extension with restricted assignment to extension points	Tests whether a targeted seasonal extension can avoid degrading states already covered by the base library.

The comparison treats library construction as an application-dependent design choice. A library that minimizes average errors over the full time series may not minimize the upper tail of the error distribution, and a library designed to cover extreme states may not be the most accurate for typical states. For this reason, the subsequent evaluation treats the library construction problem as a multi-criteria design choice and reports results separately for the three target quantities and for typical, upper-tail, critical-state, post-action, and runtime-related criteria.

4. Data Basis, Study Design, and Evaluation Protocol

This section defines the empirical basis and validation protocol used to evaluate the operating-point libraries introduced in Section 3. The empirical basis is a reproducible SimBench benchmark. All comparisons use the same nonlinear AC power-flow reference and the same three monitored target quantities: voltage magnitude, line loading, and transformer loading. The section also specifies the accuracy metrics, the definition of critical states, the post-action validation protocol, and the runtime measurements used in Section 5.

4.1. Overall Evaluation Workflow

The evaluation is organized as a sequence of complementary tests rather than as a single aggregate score. First, an equal-budget comparison fixes the number of linearization points at 36 and compares alternative operating-point selection strategies. Second, a point-count analysis varies the number of k-medoids representatives between 10, 22, 36, and 50 to quantify the marginal benefit of larger libraries. Third, a controlled hybrid ablation keeps a 28-point k-medoids basis fixed and compares three strategies for adding eight targeted points. Fourth, the methods are evaluated in operating states close to voltage and loading limits. Fifth, large line-loading errors outside the near-limit region are analyzed separately to distinguish numerical error magnitude from operational relevance. Sixth, selected actions derived from the linear models are checked by nonlinear AC power flows. Finally, the computational effort is measured in a controlled runtime benchmark.

This workflow separates three aspects that would otherwise be conflated: the statistical accuracy of a library over a representative validation sample, its behavior in safety-relevant states, and its computational cost. Voltage magnitude, line loading, and transformer loading are therefore reported as separate target quantities throughout the study. No global ranking or combined score across these quantities is formed.

4.2. Benchmark Grids and Validation Sample

The analysis uses five distribution grids from the SimBench dataset, which provides reproducible benchmark models and standardized time-series data for power-flow studies [30]. The selected grid set covers rural, semi-urban, and urban low-voltage networks as well as rural and urban combined medium-/low-voltage systems. This selection is intended to expose the linearized models to different topology classes, voltage levels, and loading patterns while retaining a controlled and reproducible benchmark setting.

Table 3 summarizes the five benchmark networks and their common validation basis. For each network, all evaluated operating-point libraries are assessed on a common set of 500 validation time steps. The same validation states are used for all methods within a network, and cross-network summaries assign equal weight to each grid. Equal network weighting is used to compare method behavior across grid types rather than to reproduce the frequency with which individual network classes occur in practice.

Table 3. Benchmark networks and common validation basis used in the final evaluation.

Network Identifier	Grid Class	Validation States	Role in the Evaluation
1-LV-rural1–2-sw	Low-voltage rural	500	Rural low-voltage reference with strong local feed-in and loading variation.
1-LV-semiurb4–2-sw	Low-voltage semi-urban	500	Intermediate low-voltage structure between rural and urban operating patterns.
1-LV-urban6–2-sw	Low-voltage urban	500	Urban low-voltage reference with denser load distribution.
1-MVLV-rural-2.107-2-sw	Combined MV/LV rural	500	Larger rural benchmark used for voltage-dominated critical states and post-action cases.
1-MVLV-urban-5.303-2-sw	Combined MV/LV urban	500	Larger urban benchmark and near-limit control case in the post-action validation.

4.3. AC Reference and Monitored Target Quantities

Each validation state is evaluated by a nonlinear AC power-flow calculation using the same network topology and nodal injections as the corresponding linearized model. The AC result serves as the reference for all error metrics and all constraint classifications. The monitored output vector is partitioned into three target quantities,

$$y_t = (v_t, \ell_t^{\text{line}}, \ell_t^{\text{trafo}}), \quad (30)$$

where v_t contains voltage magnitudes in per unit, ℓ_t^{line} contains line loadings in percent, and ℓ_t^{trafo} contains transformer loadings in percent. Linearized estimates are obtained with the library-assigned local model defined in Section 3. Loading errors are reported in percentage points because the monitored values are percentages of the corresponding equipment rating.

The technical limits used for constraint classification are $0.95 \leq v \leq 1.05$ p.u. for voltage magnitude and $\ell \leq 100\%$ for line and transformer loading. The voltage band is a study-specific operational threshold rather than a regulatory compliance criterion. It is chosen to define a moderate validation threshold distinct from general voltage-quality requirements at public supply terminals and from connection-assessment rules for low-voltage generation systems [31,32]. These limits define the classification of actual violations in the AC reference and predicted violations in the linearized models.

4.4. Accuracy Metrics and Constraint Classification

For a target quantity q , method r , time step t , and monitored element i , the signed approximation error is defined as

$$e_{q,t,i}^{(r)} = \hat{y}_{q,t,i}^{(r)} - y_{q,t,i}^{\text{AC}}, \quad (31)$$

and the corresponding absolute error is

$$a_{q,t,i}^{(r)} = |e_{q,t,i}^{(r)}|. \quad (32)$$

Typical accuracy over a validation set T is summarized by the mean absolute error,

$$\text{MAE}_q^{(r)} = \frac{1}{|T|} \sum_{t \in T} \frac{1}{n_q} \sum_{i=1}^{n_q} a_{q,t,i}^{(r)}, \quad (33)$$

where n_q denotes the number of monitored components of target quantity q . Time-step-specific root-mean-square errors are computed as

$$\text{RMSE}_{q,t}^{(r)} = \sqrt{\frac{1}{n_q} \sum_{i=1}^{n_q} \left(e_{q,t,i}^{(r)} \right)^2}, \quad (34)$$

and their upper tail is reported by the empirical 95th percentile,

$$\text{p95RMSE}_q^{(r)} = Q_{0.95} \left(\left\{ \text{RMSE}_{q,t}^{(r)} : t \in T \right\} \right). \quad (35)$$

Maximum absolute errors are used only as diagnostic worst-case indicators,

$$\text{MaxAE}_q^{(r)} = \max_{t \in T, i} a_{q,t,i}^{(r)}. \quad (36)$$

The main tables report MAE and p95 time-step RMSE because these metrics distinguish typical performance from upper-tail behavior without reducing the three target quantities to one combined score.

Constraint classification is evaluated by comparing binary AC and linear indicators. For a monitored component, let $I_{q,t,i}^{\text{AC}} = 1$ denote an actual violation in the AC reference and $I_{q,t,i}^{(r)} = 1$ a violation predicted by method r . A false-safe classification is an actual violation not predicted by the linear model,

$$I_{q,t,i}^{\text{AC}} = 1, \quad I_{q,t,i}^{(r)} = 0, \quad (37)$$

whereas a false alarm is a predicted violation that is not present in the AC reference,

$$I_{q,t,i}^{\text{AC}} = 0, \quad I_{q,t,i}^{(r)} = 1. \quad (38)$$

False-safe classifications are particularly relevant for operational screening because they correspond to undetected constraint violations.

4.5. Evaluation Modules

The final evaluation uses the library variants listed in Table 2. The equal-budget comparison includes k-medoids 36, k-center 36, seasonal 36, the seasonal hybrid with 28 + 8 points, and the k-medoids/k-center hybrid with 28 + 8 points. The point-count analysis is restricted to k-medoids libraries with $k = 10, 22, 36$, and 50. The hybrid ablation uses a common 28-point k-medoids basis and adds eight operating points with three extension rules: seasonal extension, k-center extension, and gated seasonal extension. A newly selected 36-point k-medoids library is included as an equal-size comparison in the hybrid analysis, but it is not part of the controlled 28 + 8 ablation.

Critical and near-limit states are defined from the AC reference. The primary near-limit threshold is 0.01 p.u. for voltage magnitude and 10 percentage points for line and transformer loading. Thus, voltage states are included when a voltage violation is present or when the AC voltage is within 0.01 p.u. of a voltage limit. Loading states are included when a loading violation is present or when the AC loading is within 10 percentage points of the 100% limit. Additional threshold sensitivities are documented in the Supplementary Material.

Large line-loading errors outside the near-limit region are evaluated separately. For this diagnostic analysis, selected events must remain at least 20 percentage points below the line-loading limit in the AC reference. This separation avoids interpreting large numerical

errors in clearly non-critical states as if they had the same operational relevance as errors close to a technical limit.

4.6. Post-Action Validation Protocol

The post-action validation is an exploratory, case-based consistency check of whether measures derived from the linearized models remain effective when evaluated by an AC power flow. Seven operating states are selected purposefully from the benchmark results: six core cases with voltage or transformer-loading relevance and one AC-feasible near-limit control case. Each case is evaluated for the five 36-point equal-budget methods and for two policies. The resulting evaluations support mechanistic comparison within these cases; their purposeful selection precludes statistical inference about general success probabilities.

The action is a uniform curtailment of all in-service static generators with positive active power at the selected time step. For a curtailment fraction $\alpha \in [0, 1]$, active and reactive generator injections are scaled proportionally,

$$p_{g,t}^+ = (1 - \alpha)p_{g,t}, \quad q_{g,t}^+ = (1 - \alpha)q_{g,t}. \quad (39)$$

This one-dimensional action was chosen as a controlled and reproducible proxy that isolates differences caused by the assigned local model. Proportional scaling of $p_{g,t}$ and $q_{g,t}$ preserves each affected generator's pre-action power factor, while loads, storage units, other generators, and network parameters remain unchanged. Its intended role is limited to this controlled comparison. Spatially selective curtailment, reactive-power control, and storage dispatch have location-, state-, and constraint-dependent effects and can therefore lead to different actions and method comparisons.

The nominal policy targets the technical limits directly. The buffered policy uses fixed illustrative margins of 0.002 p.u. below the upper voltage limit and 2 percentage points below the loading limits, corresponding to an upper-voltage target of 1.048 p.u. and loading targets of 98%. These non-normative values define common analysis variants; they were not statistically calibrated and do not provide a formal feasibility guarantee. For operational use, margins should instead be calibrated on data that are separate from the subsequent validation set. For a target quantity q , one-sided high quantiles of the signed residuals can be used, for example,

$$m_{q,\uparrow} = Q_\gamma(\max\{0, y_q^{\text{AC}} - \hat{y}_q\}), \quad m_{q,\downarrow} = Q_\gamma(\max\{0, \hat{y}_q - y_q^{\text{AC}}\}), \quad (40)$$

where the upper-limit margin covers model underestimation and the lower-limit margin covers model overestimation. The quantile level γ reflects the required conservativeness and must be validated for the intended application. Calibration should be separated by target quantity and limit direction and repeated after relevant topology or operating-distribution changes. States outside the calibrated coverage, with large assignment distance, or with actions binding at a technical limit should trigger an AC fallback calculation.

For each method and case, the smallest curtailment fraction that satisfies the linearized constraints is determined. The resulting post-action state is then evaluated by a nonlinear AC power-flow calculation,

$$y_t^{\text{AC},+} = \text{ACPF}(u_t^+), \quad (41)$$

where u_t^+ denotes the modified injection vector. The post-action validation is therefore not an optimization benchmark for a full operational control problem; it is a consistency test of whether linearized action decisions remain valid in the nonlinear reference model.

4.7. Runtime Assessment

Runtime is evaluated separately from approximation accuracy because local model libraries shift effort from repeated online evaluation to one-time preprocessing. The controlled benchmark distinguishes operating-point selection, local-model construction, model initialization, online evaluation, and AC-reference calculation. Selection and construction form the one-time offline effort. Initialization is reported separately because it depends on how the stored sensitivity matrices are loaded and prepared for repeated use. Online evaluation refers to applying an already constructed library to validation states.

Runtime measurements are performed in a common single-thread environment with fixed repetitions for selection, model construction, and online evaluation. Absolute runtimes are therefore hardware- and implementation-dependent, but comparisons within the controlled environment are meaningful. The online benefit is summarized by the speed-up ratio

$$\text{speedup} = \frac{T_{\text{AC}}}{T_{\text{lin}}}, \quad (42)$$

where T_{AC} denotes the runtime of the AC power-flow reference and T_{lin} denotes the runtime of the corresponding linearized online evaluation for the same validation states.

5. Results

The numerical evaluation is based on five SimBench distribution grids covering rural, semi-urban, and urban low-voltage networks as well as rural and urban combined medium-/low-voltage networks. Unless stated otherwise, each method is evaluated on the same set of 500 validation time steps per network, and cross-network means assign equal weight to each grid. The results are presented in seven steps: the equal-budget comparison of operating-point selection strategies, the influence of library size, the controlled hybrid ablation, performance in critical and near-limit operating states, a separate diagnostic analysis of large line-loading errors outside the near-limit region, post-action validation, and the runtime–accuracy trade-off.

5.1. Equal-Budget Comparison of Operating-Point Selection Strategies

The first comparison fixes the library size at 36 linearization points and therefore isolates the effect of the operating-point selection strategy. The evaluated methods comprise k-medoids, greedy k-center selection, the seasonal selection, and two hybrid variants that retain a common set of 28 k-medoids and add eight targeted points. Table 4 reports the cross-network mean errors for voltage magnitude, line loading, and transformer loading.

k-Medoids 36 attains the first target-specific rank for all three target quantities. Its p95 time-step RMSE is 1.148×10^{-4} p.u. for voltage magnitude, 0.658 percentage points for line loading, and 1.044 percentage points for transformer loading. The k-medoids/k-center hybrid ranks second for all three targets and remains close to k-Medoids 36 for voltage and line loading. It yields a marginally lower voltage MAE, 2.107×10^{-5} p.u. compared with 2.124×10^{-5} p.u., but a higher voltage p95 RMSE. For transformer loading, k-Medoids 36 retains a clearer advantage in both reported metrics. Figure 1 expresses the same comparison relative to k-Medoids 36.

The zero reference is used only to improve readability; k-Medoids 36 remains a full candidate in the comparison. The two hybrid variants lie substantially closer to the reference than the pure seasonal and pure k-center selections. The latter two methods show higher errors for most of the six reported metrics and do not provide a compensating advantage in repeated online evaluation, as shown in Section 5.7.

The equal-budget results therefore indicate that statistical representativeness over the observed operating range is more effective as a general-purpose selection objective than

either seasonal coverage alone or a pure geometric covering objective. At the same time, the close performance of the k-medoids/k-center hybrid motivates a more targeted analysis of extension points.

Table 4. Cross-network mean approximation errors for five operating-point selection methods at an equal budget of 36 linearization points. Best values are shown in bold and second-best values are underlined.

Method	Voltage Magnitude		Line Loading		Transformer Loading		Ranks V/L/T –
	MAE	p95 RMSE	MAE	p95 RMSE	MAE	p95 RMSE	
	10^{-5} p.u.	10^{-4} p.u.	pp	pp	pp	pp	
k-Medoids 36	<u>2.124</u>	1.148	0.083	0.658	0.210	1.044	1/1/1
k-center 36	3.094	1.606	0.135	0.914	0.354	1.579	4/5/5
Seasonal 36	3.541	1.917	0.131	1.041	0.326	1.613	5/4/4
Seasonal hybrid (28 + 8)	2.155	<u>1.190</u>	0.084	0.694	<u>0.230</u>	1.125	3/3/3
k-center hybrid (28 + 8)	2.107	1.195	<u>0.083</u>	<u>0.682</u>	0.231	<u>1.112</u>	2/2/2

Loading errors are reported in percentage points (pp). The rank triplet is target-specific; no cross-target global ranking is formed. The k-center hybrid denotes a 28-point k-medoids basis extended by eight k-center points.

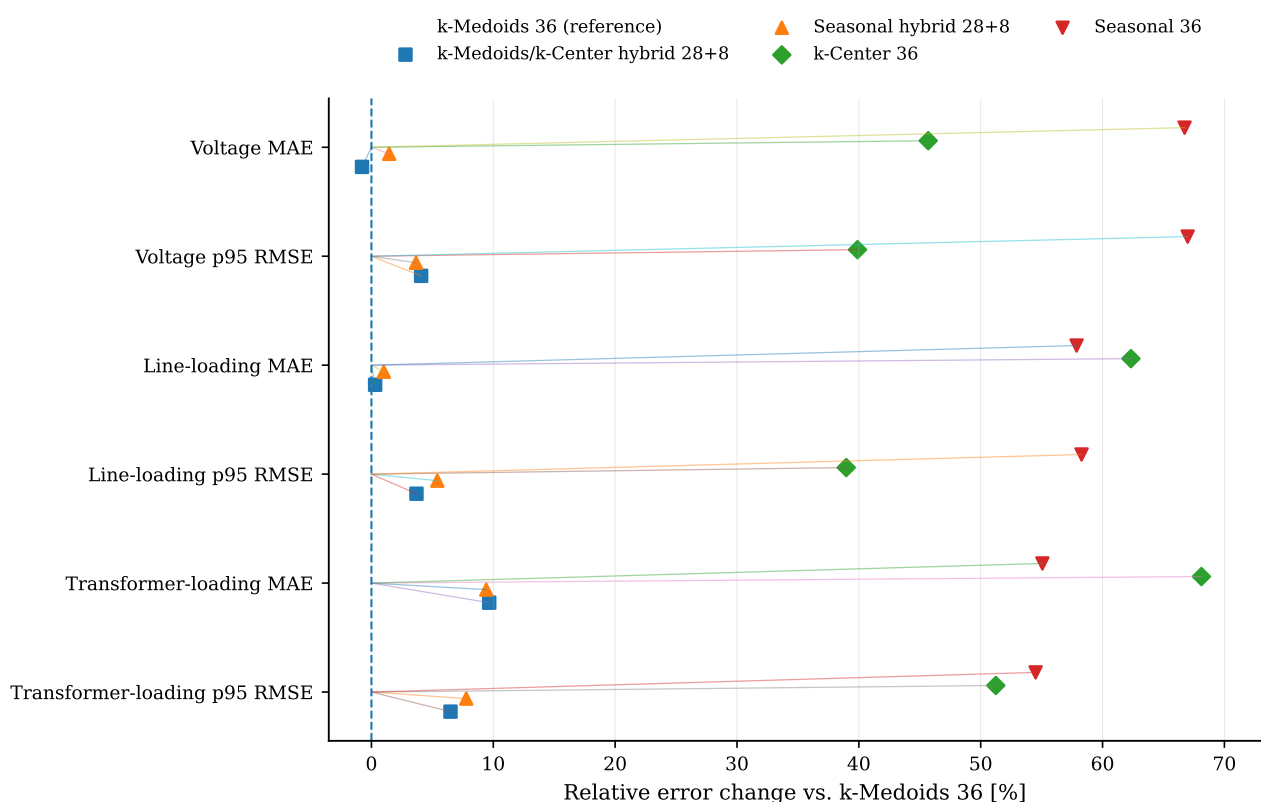


Figure 1. Relative change in the cross-network mean approximation errors at an equal budget of 36 linearization points. k-Medoids 36 is shown explicitly and serves as the internal zero reference because it achieved the strongest target-specific equal-budget performance. Negative values indicate lower errors and positive values indicate higher errors. The k-medoids/k-center hybrid remains close to the reference across the six metrics, whereas the k-center and seasonal selections exhibit substantially larger errors.

5.2. Influence of the Number of Linearization Points

The second analysis varies the k-medoids library size between $k = 10, 22, 36$, and 50 . Because the libraries are selected independently for each value of k , they are not nested.

Consequently, increasing the library size does not mathematically guarantee that every error metric decreases monotonically. Panel A of Table 5 and Figure 2 report the marginal error reductions between successive library sizes. The transition from 10 to 22 points produces the largest and most consistent improvement: the six highlighted metrics decrease by 21.7% to 29.0%. The transition from 22 to 36 points yields smaller reductions of 5.1% to 12.1% for five metrics, while the transformer-loading p95 RMSE increases slightly by 1.4%. Increasing the library from 36 to 50 points provides further reductions of 4.2% to 19.6%, with the largest gains occurring for transformer loading. Across the non-maximum metrics, the 36-point library captures on average 80.4% of the total error reduction achieved between 10 and 50 points, while using 28% fewer linearization points than the 50-point library. This result supports the use of $k = 36$ as a common comparison budget rather than as a universally optimal library size. The limited monotonicity of individual maximum errors further shows that the number of points alone is not sufficient to characterize a library; the locations of the selected operating points remain decisive.

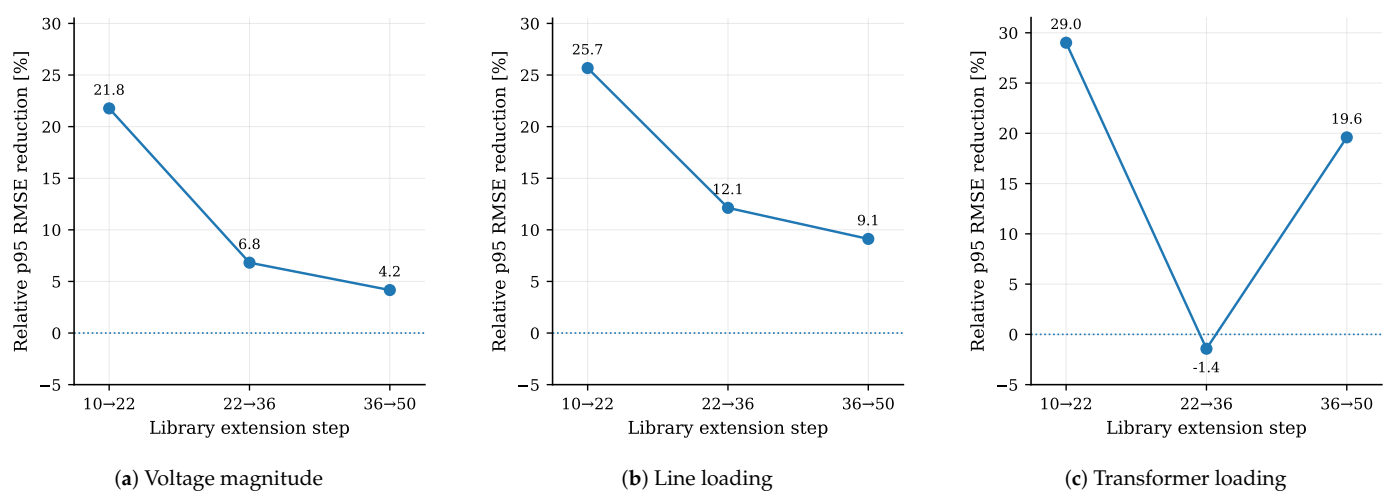


Figure 2. Marginal cross-network accuracy gains of increasing the k-medoids library size from 10 to 22, from 22 to 36, and from 36 to 50 linearization points for (a) voltage magnitude, (b) line loading, and (c) transformer loading. The ordinate reports the percentage reduction of the p95 time-step RMSE relative to the immediately preceding library size. The largest gain occurs at the first extension step, and the additional benefit decreases thereafter; for transformer loading, the transition from 22 to 36 points yields a slight deterioration.

Table 5. Effect of library size and targeted extension points. Positive values in Panel A indicate error reductions; negative values indicate deterioration. Best values in Panel B are shown in bold and second-best values are underlined.

Panel A: Marginal error reduction (%)						
Transition	Voltage magnitude		Line loading		Transformer loading	
	MAE	p95 RMSE	MAE	p95 RMSE	MAE	p95 RMSE
10–22	27.9	21.8	21.7	25.7	26.4	29.0
22–36	6.5	6.8	5.7	12.1	5.1	−1.4
36–50	7.1	4.2	6.7	9.1	14.9	19.6

Table 5. Cont.

Panel B: Hybrid ablation							
Method	<i>k</i>	Voltage magnitude		Line loading		Transformer loading	
		MAE 10 ^{−5} p.u.	p95 RMSE 10 ^{−4} p.u.	MAE PP	p95 RMSE PP	MAE PP	p95 RMSE PP
k-Medoids 28	28	2.196	1.211	0.085	0.706	0.233	1.137
Seasonal hybrid (28 + 8)	36	2.155	1.190	0.084	0.694	0.230	1.125
k-center hybrid (28 + 8)	36	2.107	1.195	0.083	0.682	0.231	1.112
Gated seasonal (28 + 8)	36	2.166	1.198	0.084	0.698	0.231	1.128
k-Medoids 36	36	2.124	1.148	0.083	0.658	0.210	1.044

The k-medoids libraries for different values of *k* are not nested. Panel B compares the common 28-point basis, three targeted 28 + 8 extensions, and a newly selected 36-point k-medoids library. The k-center hybrid denotes the k-center extension of the common basis.

5.3. Targeted Extension and Hybrid Ablation

The hybrid ablation holds the initial 28 k-medoids fixed and compares three strategies for adding eight operating points: seasonal extension, k-center extension, and a gated seasonal extension. Panel B of Table 5 shows that all three targeted extensions improve all six primary accuracy metrics relative to the common 28-point basis.

The k-medoids/k-center hybrid is the most consistent targeted extension. Relative to k-Medoids 28, it reduces the voltage MAE by 4.0%, the line-loading MAE by 2.1%, the line-loading p95 RMSE by 3.3%, and the transformer-loading p95 RMSE by 2.2%. The seasonal hybrid yields the lowest voltage p95 RMSE and transformer-loading MAE among the three targeted extensions, whereas the gated seasonal variant provides smaller improvements throughout. Figure 3 makes the ablation basis explicit. The newly selected k-Medoids 36 library delivers the largest improvement for five of the six metrics, particularly for transformer loading, whereas the k-medoids/k-center hybrid yields the lowest voltage MAE. Thus, targeted additions can improve an existing library without replacing its basis, but jointly reselecting all 36 points remains more effective when the objective is broad cross-network accuracy.

5.4. Critical and Near-Limit Operating States

The general error comparison is complemented by an event-based evaluation close to technical limits. The primary near-limit thresholds are 0.01 p.u. for voltage magnitude and 10 percentage points for line and transformer loading. Table 6 summarizes the resulting event set and method-specific performance. The event set contains 4729 voltage events, including 1248 actual voltage-limit violations, and 38 transformer-loading events, including 35 actual overloads.

For voltage magnitude, the k-medoids/k-center hybrid achieves the smallest upper-tail and maximum errors, with a p95 absolute error of 1.743×10^{-4} p.u. and a maximum error of 3.401×10^{-4} p.u. k-Medoids 36 yields the second-lowest p95 error, but a larger maximum error of 6.873×10^{-4} p.u. The seasonal selection has the largest p95 and maximum errors and produces the only false-safe voltage classification. It also produces 20 false alarms, compared with 10 to 13 for the other methods.

The empirical cumulative distributions in Figure 4 show that the advantage of the k-medoids/k-center hybrid is concentrated in the upper part of the voltage-error distribution. k-Medoids 36 remains competitive over much of the distribution, whereas the seasonal curve is shifted to larger errors. The result should therefore be read as an upper-tail improvement rather than event-wise dominance: adding geometrically distant points to a statistically representative basis improves coverage of the critical upper tail.

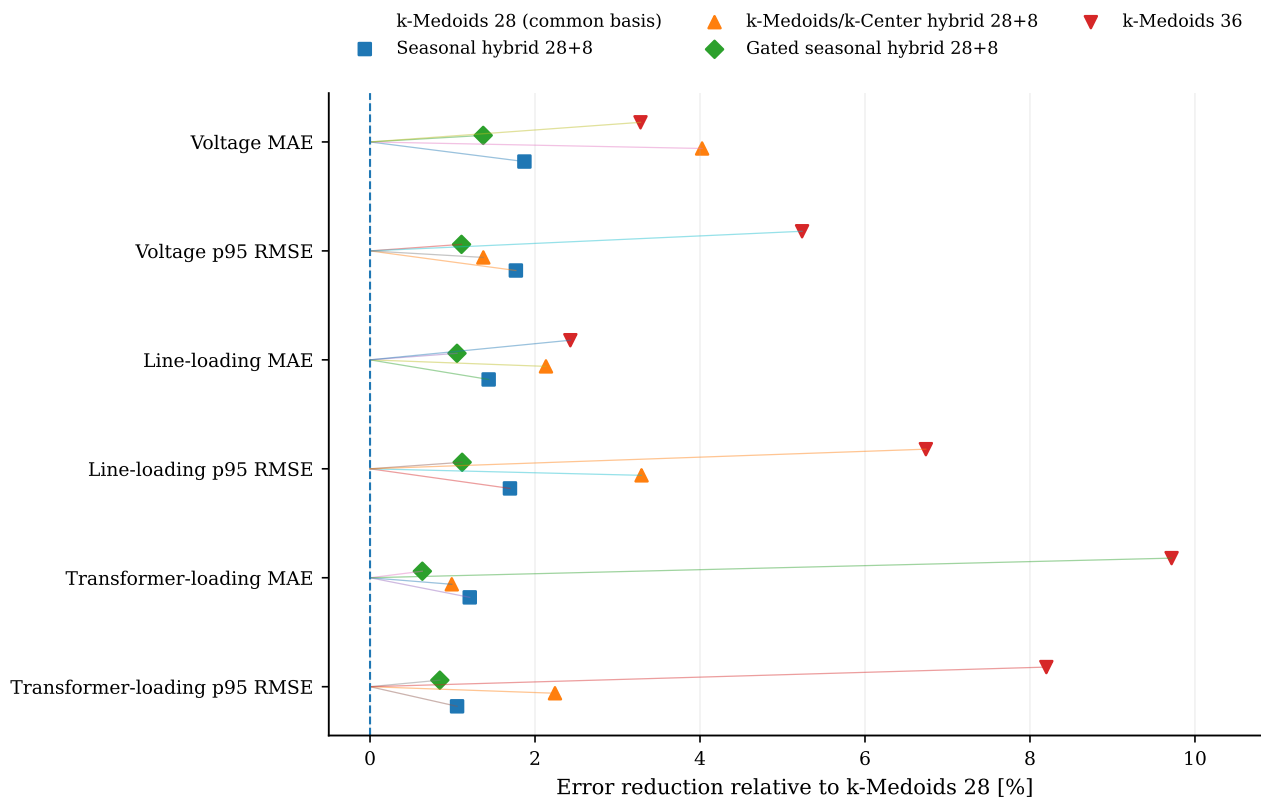


Figure 3. Cross-network error reduction relative to the common 28-point k-Medoids basis. Positive values indicate improvement. The k-center extension provides the strongest targeted hybrid improvement across the six metrics, whereas the jointly selected 36-point k-Medoids library remains superior for several upper-tail and transformer-loading metrics.

Table 6. Performance for critical and near-limit events using the primary thresholds of 0.01 p.u. for voltage and 10 percentage points for loading. Best values are shown in bold, and second-best values are underlined.

Panel A: Voltage (4729 critical events; 1248 actual violations)				
Method	p95 error 10^{-4} p.u.	Max. error 10^{-4} p.u.	False-safe count	False-alarm count
k-Medoids 36	<u>2.123</u>	6.873	0	11
k-center 36	2.154	5.213	0	11
Seasonal 36	7.140	11.882	1	20
Seasonal hybrid (28 + 8)	2.315	<u>4.657</u>	0	13
k-center hybrid (28 + 8)	1.743	3.401	0	10
Panel B: Transformer loading (38 critical events; 35 actual violations)				
Method	p95 error PP	Max. error PP	False-safe count	False-alarm count
k-Medoids 36	0.076	0.134	0	0
k-center 36	0.072	0.080	0	0
Seasonal 36	0.228	0.287	0	0
Seasonal hybrid (28 + 8)	0.101	0.134	0	0
k-center hybrid (28 + 8)	<u>0.076</u>	<u>0.103</u>	0	0

A false-safe event is an actual violation not detected by the linear model. All 35 actual transformer overloads were detected by every method. No line-loading events were within 10 percentage points of the limit. The k-center hybrid denotes a 28-point k-medoids basis extended by eight k-center points.

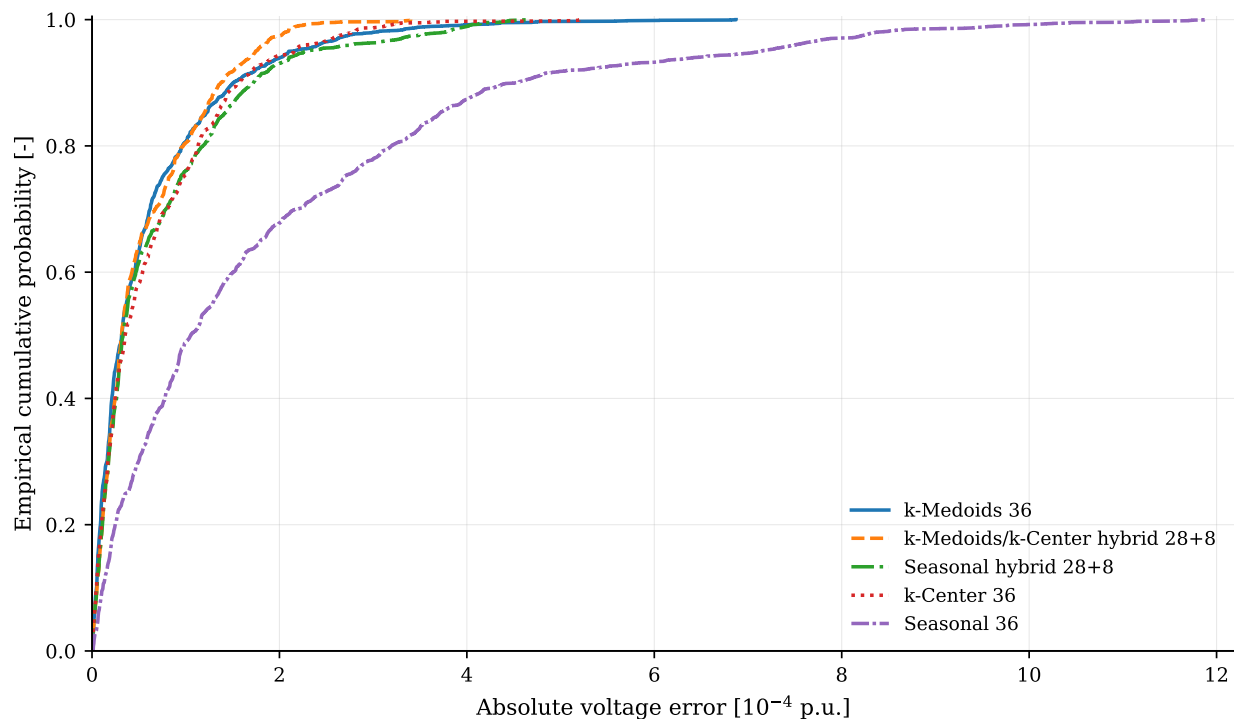


Figure 4. Empirical cumulative distribution of the absolute voltage error for the 4729 critical voltage events. Curves located further to the left indicate smaller errors over a larger share of the evaluated events. The k-medoids/k-center hybrid reduces the upper part of the error distribution, whereas the seasonal selection produces the largest errors and the only false-safe voltage classification.

All 35 actual transformer overloads are detected by every method, and no false transformer alarms occur. The p95 transformer-loading errors range from 0.072 to 0.228 percentage points. No line-loading event lies within 10 percentage points of the thermal limit in the common validation sample. Large line-loading errors are therefore analyzed separately from the near-limit event set.

The event counts are strongly target-dependent: 4729 voltage events support a broad voltage-focused comparison, whereas the 38 transformer events originate from one network and permit only descriptive conclusions for the observed sample. Because no line-loading event lies within 10 percentage points of the limit, the benchmark provides no evidence about near-limit line-violation detection.

5.5. Large Line-Loading Errors Outside the Near-Limit Region

A separate diagnostic analysis selects 150 line-loading events, 30 per network, with comparatively large approximation errors while remaining at least 20 percentage points below the loading limit. In fact, the AC-reference loadings of the selected events range only from 3.54% to 31.41%, corresponding to distances of 68.59 to 96.46 percentage points from the limit. Across the five methods, the largest absolute error observed for an individual selected event reaches 20.17 percentage points, but neither the AC reference nor any linear estimate approaches a thermal violation.

Across the resulting 750 method–event evaluations, 90.9% of the signed errors correspond to underestimation of line loading. The selected events involve 56 different lines, and the most frequently affected line occurs in nine selected time steps. The Spearman correlation between assignment distance and the maximum line-loading error is positive for every network–method combination, ranging from 0.323 to 0.626. Assignment distance therefore contains information about approximation difficulty, but it does not determine the error uniquely.

These findings separate numerical error magnitude from operational relevance. A large loading error can be diagnostically important even when it cannot change the limit classification. Accordingly, the near-limit analysis and the non-near-limit outlier analysis address different failure modes and should not be combined into a single ranking criterion.

5.6. Post-Action Validation

The exploratory post-action consistency check assesses whether measures derived from the linear models remain effective in the nonlinear AC power flow. This step is particularly relevant for flexibility-enabled operational optimization, where linear models may be used to screen or derive corrective actions before the resulting state is checked against nonlinear grid physics. Uniform curtailment serves as a controlled proxy that keeps the action definition identical across methods and thereby isolates model-induced differences; its scope excludes a complete market, regulatory, or dispatch framework. Seven purposefully selected operating states are evaluated with all five methods under two policies. All 70 linear action problems are feasible and predict a technically admissible post-action state. Table 7 summarizes the corresponding AC validation results. These counts describe the selected cases and must not be interpreted as statistical success rates.

Table 7. AC validation of uniformly applied curtailment measures for seven selected operating states. Ranges represent the five methods.

Case	Grid	t	Binding Limit	Pre-Action State	Nominal		With Safety Margin	
					Curtailment %	AC Success Count	Additional Curtailment %	AC Success Count
A	MV/LV rural	27,089	Upper-voltage, bus 15	$V_{\max} = 1.0772$ p.u.; 2 violations	59.46–60.74	5/5	4.39	5/5
B	MV/LV rural	32,121	Upper-voltage, bus 15	$V_{\max} = 1.0738$ p.u.; 2 violations	49.06–50.18	5/5	4.13	5/5
C	MV/LV rural	27,112	Upper-voltage, bus 15	$V_{\max} = 1.0780$ p.u.; 24 violations	58.28–58.56	5/5	4.17	5/5
D	MV/LV rural	5368	Upper-voltage, bus 15	$V_{\max} = 1.0505$ p.u.; 2 violations	1.63–2.04	1/5	8.43	5/5
E	LV rural	13,305	Transformer 0	$T_{\max} = 144.59\%$; 5 violations	43.96–44.83	5/5	1.98	5/5
F	LV rural	14,256	Transformer 0	$T_{\max} = 202.11\%$; 12 violations	70.55–70.89	5/5	1.38	5/5
G	MV/LV urban	10,214	No action required	No violation; $V_{\max} = 1.0451$ p.u.	0.00–0.00	5/5	0.00	5/5

t denotes the time-step index. The additional curtailment is reported in percentage points of the available generation. Nominal actions were successful in 31 of 35 AC evaluations; all 35 evaluations with an additional safety margin were successful.

The nominal actions are successful in 31 of 35 AC evaluations. All four remaining violations occur in case D, which is only slightly infeasible before the action, with an AC maximum voltage of 1.050467 p.u. at bus 15. The method-specific outcome is explained primarily by the signed pre-action error. k-Center overestimates the initial voltage by 1.947×10^{-5} p.u. and therefore applies 2.042% curtailment, reaching an AC post-action voltage of 1.049981 p.u. The other methods underestimate the initial voltage by 2.299×10^{-5} to 8.453×10^{-5} p.u., apply 1.630% to 1.868% curtailment, and leave AC post-action voltages between 1.050022 and 1.050079 p.u. The absolute errors in the predicted voltage change remain below 5.447×10^{-6} p.u. for all five methods. Thus, the small positive k-center bias acts as an incidental reserve specific to this upper-voltage case. The same bias can cause unnecessary action or be unfavorable for another limit direction. Figure 5 compares the nominal and buffered results for case D.

With the illustrative margin, all five methods satisfy the technical AC limit. Across all seven cases, the margin increases the curtailment fraction by 1.38 to 8.43 percentage points, depending on the case, and all 35 buffered actions are successful. No new constraint violation is introduced, and the already feasible control case G triggers no action under either policy. The four nominal failures are caused primarily by small pre-action voltage errors rather than by an incorrect predicted direction of the curtailment response. These selected cases show that targeting a technical limit exactly is fragile in a marginally infeasible state. They do not validate the fixed margins for other data and provide no formal AC-feasibility guarantee.

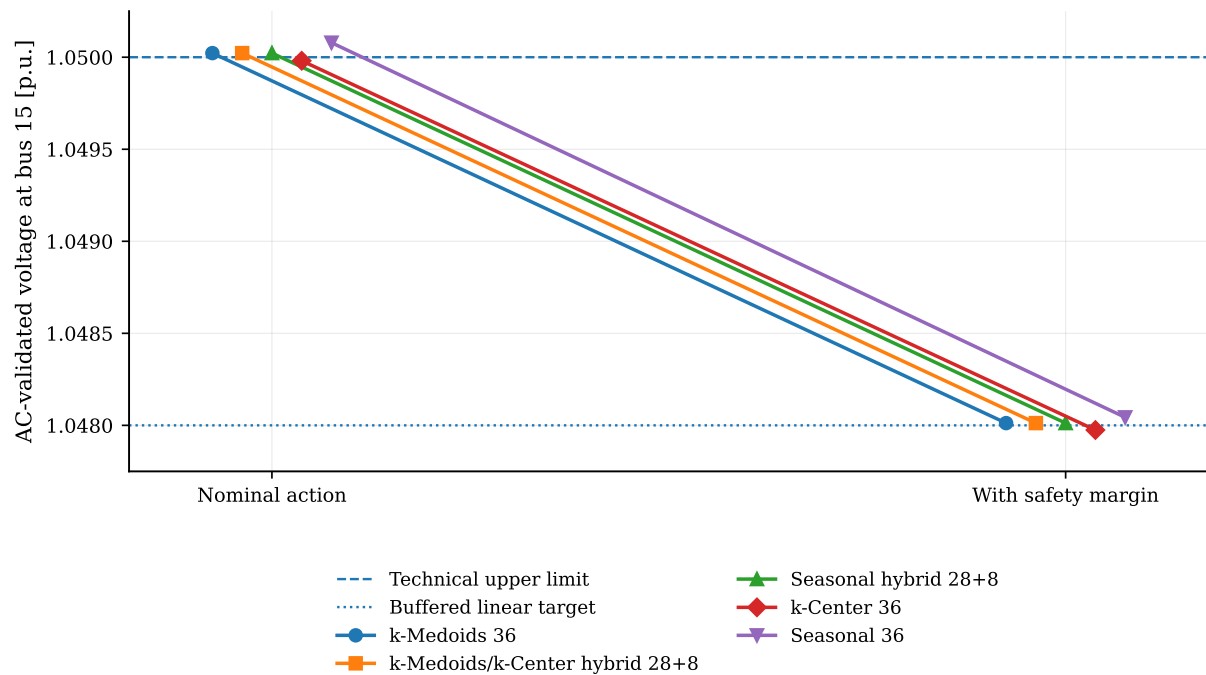


Figure 5. AC-validated voltage at bus 15 after the nominal curtailment action and after the action including an additional safety margin for post-action case D. Four of the five nominal actions remain slightly above the technical upper limit, whereas all actions with the safety margin remain below the limit.

5.7. Runtime–Accuracy Trade-Offs

The runtime analysis separates operating-point selection, local-model construction, initialization, and repeated online evaluation. Table 8 reports cross-network medians measured in a controlled single-thread environment. Absolute values are hardware- and implementation-specific; the comparison within the common environment is the relevant result.

For the k-medoids libraries, the one-time offline effort increases from 7.34 s at $k = 10$ to 40.62 s at $k = 50$, while storage grows from 3.10 to 15.49 MB and initialization from 19.55 to 97.11 s. In contrast, the online evaluation remains nearly constant at approximately 3.5 ms per state. The median speed-up over the AC power flow ranges from 26.3 to 27.9 across all methods. Thus, the number of linearization points primarily affects preprocessing, initialization, and storage, but not the repeated evaluation time at the investigated library sizes.

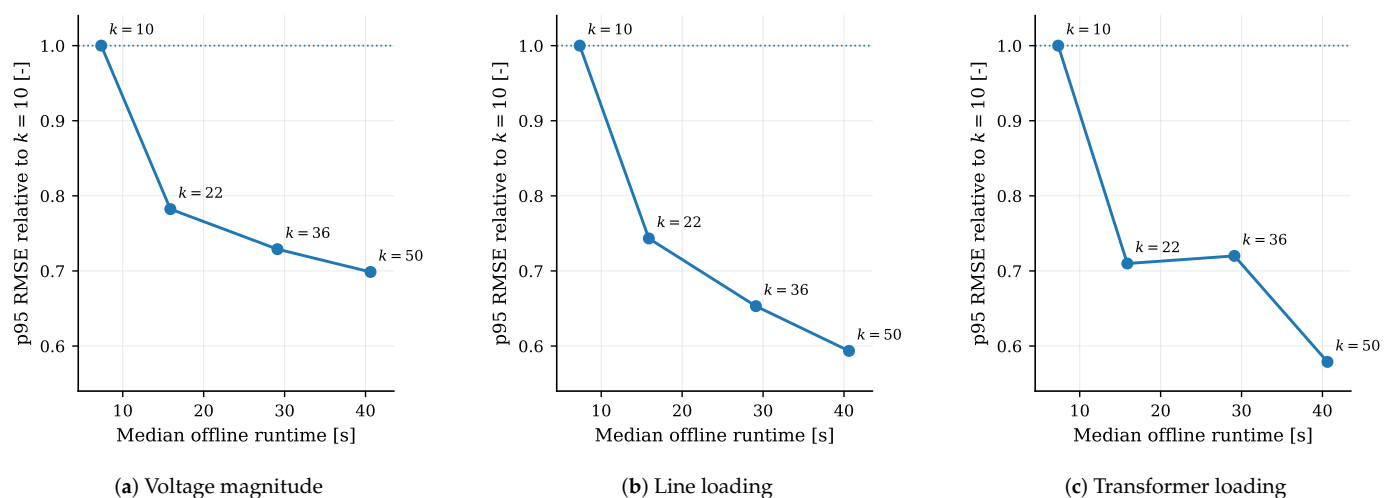
At an equal budget of 36 points, the k-medoids/k-center hybrid has an offline median of 16.12 s, compared with 18.49 s for k-Center 36 and 29.12 s for k-Medoids 36. The seasonal and seasonal-hybrid variants require 82.00 and 79.51 s, respectively, because their operating-point selection dominates the offline effort. These differences do not translate into an online speed advantage.

Table 8. Cross-network median runtime and storage results. Model initialization is reported separately from the offline total; serialization is excluded.

Method	<i>k</i> –	Selection s	Model Build s	Offline s	Initialization s	Online ms state ^{−1}	Speed-Up –	Storage MB
k-Medoids 10	10	0.40	6.94	7.34	19.55	3.504	26.9	3.10
k-Medoids 22	22	0.39	15.49	15.89	42.57	3.575	27.9	6.81
k-Medoids 36	36	0.41	28.70	29.12	68.70	3.503	27.3	11.15
k-Medoids 50	50	0.50	40.12	40.62	97.11	3.510	26.3	15.49
Seasonal 36	36	56.19	26.15	82.00	70.15	3.396	27.4	11.12
k-center 36	36	0.45	18.04	18.49	70.38	3.513	27.4	11.12
Seasonal hybrid (28 + 8)	36	66.36	15.69	79.51	71.91	3.584	27.8	11.10
k-center hybrid (28 + 8)	36	0.97	15.51	16.12	72.36	3.576	26.5	11.09

Absolute runtimes apply to the documented single-thread hardware and software environment. Speed-up is relative to the AC power flow on the same system. No target-independent runtime–accuracy score is formed. The k-center hybrid denotes a 28-point k-medoids basis extended by eight k-center points.

Figure 6 relates the median one-time offline effort to the target-specific p95 time-step RMSE for the k-medoids point-count study. The largest accuracy gain is obtained between 10 and 22 points. Beyond 22 points, additional accuracy requires progressively higher offline effort and storage, although the precise gain remains target-specific. k-Medoids 50 provides the lowest cross-network mean errors among the four library sizes, whereas $k = 36$ represents a balanced common budget for the method comparison. No target-independent combined score is formed.

**Figure 6.** Target-specific runtime–accuracy trade-off for the k-medoids point-count study: (a) voltage magnitude, (b) line loading, and (c) transformer loading. The p95 time-step RMSE is normalized to the result for $k = 10$; the horizontal axis shows the median one-time offline runtime. The shared axis ranges permit a direct comparison of the marginal accuracy gains.

Taken together, the results identify two complementary design principles. Statistical operating-range representation provides the most stable general-purpose accuracy, while targeted geometric extensions improve critical upper-tail behavior and can yield favorable accuracy at lower offline cost. The appropriate library design therefore depends on whether the application prioritizes broad time-series accuracy, critical-state coverage, or minimal one-time construction effort.

6. Discussion

The benchmark shows that the usefulness of operating-point-dependent linearized grid models depends on two linked choices: how the operating range is represented and how the resulting local models are used. No library is uniformly superior under every criterion. Instead, the results show consistent strengths for different library designs: k-medoids provides the most stable general-purpose accuracy, while k-center points are valuable when added as targeted coverage for difficult or critical regions. The number of linearization points defines the available model resolution, but the selected locations and the assignment rule determine how this resolution is used.

6.1. Library Design and Operating-Range Representation

At an equal budget of 36 linearization points, k-medoids achieves the first target-specific rank for voltage magnitude, line loading, and transformer loading (Table 4 and Figure 1). This empirically supports minimizing average assignment distance as a useful default for the investigated data, without establishing theoretical minimization of any electrical-output error. The Euclidean net-active-power distance is target-independent, whereas the mapping from an injection deviation to voltage or loading depends on $H^{(h)}$; injection-space and electrical-response proximity therefore remain distinct. Sensitivity-weighted or target-specific distances could align library construction more directly with particular voltage or loading objectives and would constitute a separate library-design problem.

The pure k-center library follows a different objective by reducing the largest assignment distance. As a stand-alone equal-budget library, this coverage-oriented strategy is less accurate than k-medoids for most reported metrics. Its value becomes visible in the targeted extension setting. Adding eight k-center points to a common 28-point k-medoids basis yields the best upper-tail and maximum voltage errors in the critical-event analysis. Thus, k-center selection is most useful here as a complement to a representative basis rather than as a replacement for it.

The structured seasonal selection is transparent and easy to communicate, but it performs less favorably in the broad comparison and in the critical voltage analysis. Calendar-based categories describe the electrical operating state only indirectly; different load and generation patterns can occur in the same category, while electrically similar states can fall into different categories. Expert-informed structures may still be useful in practice, but the results show that they require electrical validation against the actual operating-state distribution.

The point-count study further shows that more representatives do not automatically translate into proportional accuracy gains (Table 5 and Figure 2). The largest improvement occurs between 10 and 22 points, while additional gains become smaller and are not strictly monotonic because the independently selected libraries are not nested. The 36-point k-medoids library therefore provides a defensible common comparison budget for the investigated benchmark, not a universal optimum. When a complete rebuild is possible, jointly reselecting all 36 medoids remains preferable for broad accuracy; when an established library must be retained, targeted 28 + 8 extensions provide a practical refinement mechanism.

6.2. Critical States and Post-Action Validity

The critical-state results demonstrate why average time-series errors are insufficient for operational assessment (Table 6 and Figure 4). All methods produce small absolute voltage errors, but their upper-tail behavior and classification outcomes differ. The k-medoids/k-center hybrid reduces the p95 and maximum voltage errors relative to k-medoids 36,

whereas the seasonal library produces the largest upper-tail errors and the only false-safe voltage classification. This distinction matters because false-safe classifications correspond to undetected violations, while false alarms mainly trigger unnecessary caution.

For transformer loading, all 35 actual overloads are detected by every method and no false alarms occur. Given the small and concentrated sample of 38 transformer events from one network, this favorable result remains descriptive and supports no general inference about transformer-overload detection. The voltage comparison is supported by 4729 events across several networks. In contrast, the common sample contains no line-loading event within 10 percentage points of the thermal limit, so no conclusion can be drawn about near-limit line-violation detection. The line-loading outlier analysis addresses a different failure mode: numerical errors that are large but occur far from the limit. Some selected errors reach 20.17 percentage points, yet the corresponding AC-reference loadings remain far below 100%. These events are diagnostically relevant, especially because 90.9% of the signed method–event errors are underestimations, but they do not establish near-limit classification performance.

The exploratory post-action analysis adds a second operational perspective (Table 7 and Figure 5). Under the nominal policy, 31 of 35 AC evaluations are successful; all four failures occur in the same marginal voltage case. The favorable signed pre-action bias of k-center explains its case-specific success in case D. With the fixed illustrative margin, all 35 AC evaluations are successful and no new violation is introduced; the additional curtailment of 1.38 to 8.43 percentage points quantifies the cost within the selected cases. Because the seven cases were selected purposefully, these outcomes constitute a mechanistic consistency check rather than a statistical validation. In operational use, target- and direction-specific margins should be calibrated from separate residual data, and binding or insufficiently covered decisions should be verified by AC power flow.

6.3. Runtime Implications

The runtime results clarify where the computational benefit of the model-library concept arises (Table 8 and Figure 6). Once a library has been constructed and initialized, online evaluation remains close to 3.5 ms per state and provides a median speed-up of approximately 26 to 28 relative to the AC power flow. At the investigated library sizes, increasing the number of representatives mainly affects one-time preprocessing, initialization, and storage, while the repeated online cost remains nearly constant because each state is assigned to one local model.

This cost structure affects the preferred library design. If a library is built once and reused for many time steps, the higher offline effort of k-medoids 36 can be amortized, and its broad accuracy makes it an attractive default. If the operating range changes frequently or a library must be extended incrementally, the k-medoids/k-center hybrid becomes more competitive because it combines favorable critical-state behavior with lower measured offline effort than full 36-point k-medoids construction. The more transferable finding is the structure of the trade-off: online evaluation scales favorably for repeated screening and optimization workflows, whereas offline costs matter most when only few states are evaluated or the library must be rebuilt often.

For a distribution-system operator, the results suggest a two-stage workflow that retains AC calculations as an explicit safeguard. Offline, representative scenarios or measurements are used to construct the library, solve the AC reference at each representative, derive the local sensitivity matrices, calibrate target- and direction-specific residual margins on separate data, and define the assignment-distance range for which the library has been validated. Online, the current injection estimate provides both the full P/Q input vector for the local model and the net-active-power feature vector for nearest-representative

assignment. The assigned model can then support rapid screening or candidate-action evaluation. An AC fallback calculation is retained when a constraint is predicted to bind, the assignment distance exceeds the calibrated range, the operating distribution shifts materially, or topology and model parameters have changed. This proposed workflow has not yet been evaluated as a control-center integration.

6.4. Limitations and Future Work

Several limitations define the scope of these conclusions. The benchmark covers five SimBench grids and a common sample of 500 validation time steps per network. Equal network weighting supports controlled comparison across grid types, but it does not represent the practical frequency of particular networks or operating conditions. The operating-point libraries are derived and validated on states from the represented time-series distributions; abrupt topology changes, parameter errors, missing or noisy measurements, and operating regimes outside the library-design data require renewed validation and may require rebuilding the library.

The event distribution also limits the interpretation of critical-state results. Voltage conclusions are supported by 4729 events, whereas the transformer findings are descriptive results from 38 events in one network, including 35 overloads. The line-loading results contain no event within 10 percentage points of the thermal limit and therefore do not establish near-limit line-classification performance. The supplementary threshold analyses make this dependence transparent, but additional data with marginal line-loading states are needed. The post-action study is an exploratory consistency check limited to seven purposefully selected states, uniform photovoltaic curtailment, and two decision policies. It provides no statistical success probability and does not establish transferability to spatially selective curtailment, reactive-power control, storage dispatch, other networks, or unobserved operating conditions.

Finally, runtime measurements reflect one controlled single-thread implementation. Parallel model construction, approximate clustering, warm starts, or incremental updates could change absolute offline costs. Future work should therefore focus on adaptive library management: adding representatives when assignment distance and constraint proximity indicate insufficient coverage, removing rarely used models, updating libraries after topology or scenario changes, and calibrating target-specific decision margins from validated residuals. Broader validation should include additional benchmark and real distribution grids, operating regimes not used for library construction, genuinely near-limit line-loading cases, and a wider range of flexibility actions. Conservative or uncertainty-aware local models may further reduce the reliance on fixed empirical margins, while AC fallback calculations can remain an explicit safeguard for binding or out-of-distribution decisions [14,15].

Future library-design studies should also compare the current net-active-power proxy with active-/reactive-power and sensitivity-weighted feature representations. Such variants may improve alignment with target-specific electrical responses; their benefits require separate empirical evaluation for voltage, line loading, and transformer loading.

Overall, the study supports operating-point-dependent linearization as a practical model-library concept. Its usefulness results from representative local models, targeted coverage of difficult regions, application-specific validation criteria, and decision rules that explicitly account for residual local-model error.

7. Conclusions

This study evaluated how the representation of the operating range affects operating-point-dependent linearized power-flow models for active distribution grids. Five selection

strategies were compared across five SimBench networks using voltage magnitude, line loading, and transformer loading as separate target quantities. The evaluation combined equal-budget accuracy, library-size effects, targeted library extensions, critical-state performance, post-action AC validation, and controlled runtime measurements. The results establish operating-point library design as a core modeling decision for local linearized power-flow models.

At an equal budget of 36 linearization points, k-medoids provided the most consistent general-purpose performance and achieved the first target-specific rank for all three quantities. Increasing the library size produced clear but diminishing gains: the largest improvement occurred between 10 and 22 points, while the additional benefit beyond 36 points was smaller and not strictly monotonic because the independently selected libraries were not nested. The 36-point library therefore represents a balanced library size for the investigated benchmark rather than a universal optimum. Targeted extensions remained useful when an existing library had to be retained. In particular, adding eight k-center points to a common 28-point k-medoids basis improved all six primary accuracy metrics and provided the best upper-tail voltage performance in the critical-event analysis, although a jointly selected 36-point k-medoids library remained more accurate for most broad time-series metrics.

The critical-state results demonstrate that average errors alone are insufficient for operational assessment. For 4729 critical voltage events, the k-medoids/k-center hybrid reduced the p95 and maximum absolute errors to 1.743×10^{-4} and 3.401×10^{-4} p.u., respectively. The seasonal library produced the largest upper-tail errors and the only false-safe voltage classification. All 35 actual transformer overloads were detected by every method; given the 38 events from one network, this result remains descriptive. Because the common sample contained no line-loading event within 10 percentage points of the thermal limit, the study does not establish near-limit line-violation detection performance. These findings support target- and constraint-specific validation criteria rather than a single aggregate accuracy score.

The exploratory post-action analysis further showed that a model can estimate the current state accurately while remaining fragile when an action is designed to terminate exactly at a technical limit. Nominal curtailment actions were successful in 31 of 35 AC evaluations; all four failures occurred in one marginal voltage case. The isolated k-center success arose from a small favorable signed pre-action bias and offers no evidence of a generally superior action model. The fixed margin increased curtailment by 1.38 to 8.43 percentage points and yielded successful AC results in all 35 evaluations without creating new violations. These results are limited to seven purposefully selected cases and do not support statistical inference or a formal feasibility guarantee. Operational margins should be calibrated from separate one-sided residual data, and binding or insufficiently covered actions should be validated by AC power flow.

The computational results support the suitability of the model-library concept for repeated analysis. Online evaluation required approximately 3.5 ms per state and achieved a median speed-up of about 26–28 relative to the AC power flow in the controlled single-thread environment. Library size primarily affected offline construction time, initialization, and storage, while the repeated online cost remained nearly constant over the investigated range. Consequently, representative local-model libraries are most attractive when they can be reused for many time steps, screening tasks, or optimization iterations.

Overall, operating-point-dependent linearization provides a practical accuracy–runtime compromise provided that four conditions are addressed: the library represents the relevant operating distribution, difficult regions receive targeted coverage, validation focuses on operationally relevant constraints, and residual model error is accounted for in

binding decisions. For the investigated networks, k-medoids offers the strongest default library, while k-center points are valuable as targeted extensions for poorly covered or critical regions. Future work should extend the validation to additional networks and operating regimes, genuinely near-limit line-loading cases, topology and parameter changes, and further flexibility actions. Adaptive library updates and target-specific decision margins are particularly promising for translating the demonstrated model-library concept into reliable operational decision support.

Supplementary Materials: The following supporting information can be downloaded at <https://www.mdpi.com/article/10.3390/en19163793/s1>. Supplementary File S1: Supplementary Material PDF, containing Table S1: Benchmark scope and validation protocol; Table S2: Operating-point library catalogue used across the evaluation; Table S3: Reproducibility and integrity checks relevant for the submitted results; Table S4: Network-specific best libraries for p95 time-step RMSE in the equal-budget comparison; Table S5: Cross-network mean errors for the k-medoids point-count analysis; Table S6: Monotonicity diagnostics for independently selected k-medoids libraries; Table S7: Assignment rates to extension points in the hybrid libraries; Table S8: Relative improvement versus the common 28-point k-medoids basis; Table S9: Threshold sensitivity of the reference critical-event set, aggregated across the five networks; Table S10: Summary of selected non-near-limit line-loading outlier events; Table S11: Method- and policy-specific post-action AC validation results; Table S12: Additional curtailment introduced by the buffered policy relative to the nominal policy; Table S13: Network-level runtime scaling diagnostics; Table S14: Relative component shares in local-model construction; Table S15: Submitted supplementary data archives. The seven machine-readable data archives listed in Table S15 are also provided.

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References

1. Molzahn, D.K.; Hiskens, I.A. A Survey of Relaxations and Approximations of the Power Flow Equations. *Found. Trends Electr. Energy Syst.* **2019**, *4*, 1–221. [CrossRef]
2. Dall’Anese, E.; Bernstein, A. Real-Time Feedback-Based Optimization of Distribution Grids: A Unified Approach. *IEEE Trans. Control Netw. Syst.* **2019**, *6*, 1197–1209. [CrossRef]
3. Mahmoodi, M.; Noori, S.M.; Blackhall, L.; Scott, P. A Comparison on Power Flow Models for Optimal Power Flow Studies in Integrated Medium and Low Voltage Unbalanced Distribution Networks. *Sustain. Energy Grids Netw.* **2024**, *38*, 101339. [CrossRef]

4. Baran, M.E.; Wu, F.F. Optimal Capacitor Placement on Radial Distribution Systems. *IEEE Trans. Power Deliv.* **1989**, *4*, 725–734. [\[CrossRef\]](#)
5. Gan, L.; Low, S.H. Convex Relaxations and Linear Approximation for Optimal Power Flow in Multiphase Radial Networks. In *Proceedings of the 2014 Power Systems Computation Conference (PSCC)*; IEEE: Piscataway, NJ, USA, 2014; pp. 1–9. [\[CrossRef\]](#)
6. Bernstein, A.; Dall’Anese, E. Linear Power-Flow Models in Multiphase Distribution Networks. In *Proceedings of the 2017 IEEE PES Innovative Smart Grid Technologies Conference Europe (ISGT-Europe)*; IEEE: Piscataway, NJ, USA, 2017; pp. 1–6. [\[CrossRef\]](#)
7. Huang, J.; Cui, B.; Zhou, X.; Bernstein, A. A Generalized LinDistFlow Model for Power Flow Analysis. In *Proceedings of the 2021 60th IEEE Conference on Decision and Control (CDC)*; IEEE: Piscataway, NJ, USA, 2021; pp. 349–354. [\[CrossRef\]](#)
8. Bolognani, S.; Zampieri, S. On the Existence and Linear Approximation of the Power Flow Solution in Power Distribution Networks. *IEEE Trans. Power Syst.* **2016**, *31*, 163–172. [\[CrossRef\]](#)
9. Bolognani, S.; Dörfler, F. Fast Power System Analysis via Implicit Linearization of the Power Flow Manifold. In *Proceedings of the 2015 53rd Annual Allerton Conference on Communication, Control, and Computing (Allerton)*; IEEE: Piscataway, NJ, USA, 2015; pp. 402–409. [\[CrossRef\]](#)
10. Dhople, S.V.; Guggilam, S.S.; Chen, Y.C. Linear Approximations to AC Power Flow in Rectangular Coordinates. In *Proceedings of the 2015 53rd Annual Allerton Conference on Communication, Control, and Computing (Allerton)*; IEEE: Piscataway, NJ, USA, 2015; pp. 211–217. [\[CrossRef\]](#)
11. Christakou, K.; Le Boudec, J.Y.; Paolone, M.; Tomozei, D.C. Efficient Computation of Sensitivity Coefficients of Node Voltages and Line Currents in Unbalanced Radial Electrical Distribution Networks. *IEEE Trans. Smart Grid* **2013**, *4*, 741–750. [\[CrossRef\]](#)
12. Talkington, S.; Grijalva, S.; Reno, M.J.; Azzolini, J.A.; Peppanen, J. Localized Structure in Secondary Distribution System Voltage Sensitivity Matrices. *Electr. Power Syst. Res.* **2024**, *226*, 109788. [\[CrossRef\]](#)
13. Hamilton, D.; Navarro, L.; Aliprantis, D. Sensitivity-Based Voltage Constraints for Optimal Power Flow in Low-Voltage Distribution Feeders. *Sustain. Energy Grids Netw.* **2025**, *43*, 101855. [\[CrossRef\]](#)
14. Buason, P.; Misra, S.; Molzahn, D.K. A Sample-Based Approach for Computing Conservative Linear Power Flow Approximations. *Electr. Power Syst. Res.* **2022**, *212*, 108579. [\[CrossRef\]](#)
15. Buason, P.; Misra, S.; Watson, J.P.; Molzahn, D.K. Adaptive Power Flow Approximations with Second-Order Sensitivity Insights. *IEEE Trans. Power Syst.* **2025**, *40*, 2648–2660. [\[CrossRef\]](#)
16. Buason, P.; Misra, S.; Molzahn, D.K. Sample-Based Piecewise Linear Power Flow Approximations Using Second-Order Sensitivities. In *Proceedings of the 2025 IEEE Kiel PowerTech*; IEEE: Piscataway, NJ, USA, 2025. [\[CrossRef\]](#)
17. Taheri, B.; Gupta, R.K.; Molzahn, D.K. Optimizing Parameters of the LinDistFlow Power Flow Approximation for Distribution Systems. *IEEE Trans. Smart Grid* **2026**, *17*, 1029–1042. [\[CrossRef\]](#)
18. Di Fazio, A.R.; Perna, S.; Russo, M.; De Santis, M. Linear Power Flow Method for Radial Distribution Systems Including Voltage Control Devices. *IEEE Trans. Ind. Appl.* **2024**, *60*, 4749–4761. [\[CrossRef\]](#)
19. Sulaimon, W.O.; Habiballah, I.O.; Khalid, M.; Al-Ismail, F.S. Achieving Accurate and Efficient Load Flow Model in Distribution Networks Using Hybrid Linearization Approach. *IEEE Access* **2024**, *12*, 174193–174203. [\[CrossRef\]](#)
20. Poncelet, K.; Höschle, H.; Delarue, E.; Virag, A.; D’haeseleer, W. Selecting Representative Days for Capturing the Implications of Integrating Intermittent Renewables in Generation Expansion Planning Problems. *IEEE Trans. Power Syst.* **2017**, *32*, 1936–1948. [\[CrossRef\]](#)
21. Teichgraber, H.; Brandt, A.R. Clustering Methods to Find Representative Periods for the Optimization of Energy Systems: An Initial Framework and Comparison. *Appl. Energy* **2019**, *239*, 1283–1293. [\[CrossRef\]](#)
22. Kotzur, L.; Markewitz, P.; Robinius, M.; Stolten, D. Time Series Aggregation for Energy System Design: Modeling Seasonal Storage. *Appl. Energy* **2018**, *213*, 123–135. [\[CrossRef\]](#)
23. Scott, I.J.; Carvalho, P.M.S.; Botterud, A.; Silva, C.A. Clustering Representative Days for Power Systems Generation Expansion Planning: Capturing the Effects of Variable Renewables and Energy Storage. *Appl. Energy* **2019**, *253*, 113603. [\[CrossRef\]](#)
24. Teichgraber, H.; Lindenmeyer, C.P.; Baumgärtner, N.; Kotzur, L.; Stolten, D.; Robinius, M.; Bardow, A.; Brandt, A.R. Extreme Events in Time Series Aggregation: A Case Study for Optimal Residential Energy Supply Systems. *Appl. Energy* **2020**, *275*, 115223. [\[CrossRef\]](#)
25. Kaufman, L.; Rousseeuw, P.J. *Finding Groups in Data: An Introduction to Cluster Analysis*; John Wiley & Sons: New York, NY, USA, 1990. [\[CrossRef\]](#)
26. Gonzalez, T.F. Clustering to Minimize the Maximum Intercluster Distance. *Theor. Comput. Sci.* **1985**, *38*, 293–306. [\[CrossRef\]](#)
27. Buason, P.; Misra, S.; Talkington, S.; Molzahn, D.K. A Data-Driven Sensor Placement Approach for Detecting Voltage Violations in Distribution Systems. *Electr. Power Syst. Res.* **2024**, *232*, 110387. [\[CrossRef\]](#)
28. Taheri, B.; Molzahn, D.K. AC Power Flow Feasibility Restoration via a State Estimation-Based Post-Processing Algorithm. *Electr. Power Syst. Res.* **2024**, *235*, 110642. [\[CrossRef\]](#)
29. An, M.; Lu, T.; Han, X. An Online Updated Linear Power Flow Model Based on Regression Learning. *IET Gener. Transm. Distrib.* **2024**, *18*, 2006–2019. [\[CrossRef\]](#)

30. Meinecke, S.; Sarajlić, D.; Drauz, S.R.; Klettke, A.; Lauven, L.P.; Rehtanz, C.; Moser, A.; Braun, M. SimBench—A Benchmark Dataset of Electric Power Systems to Compare Innovative Solutions Based on Power Flow Analysis. *Energies* **2020**, *13*, 3290. [\[CrossRef\]](#)
31. EN 50160:2022; Voltage Characteristics of Electricity Supplied by Public Electricity Networks. CENELEC: Brussels, Belgium, 2022.
32. VDE-AR-N 4105:2018-11; Power Generating Plants Connected to the Low-Voltage Network. VDE FNN: Berlin, Germany, 2018; VDE Application Rule.

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